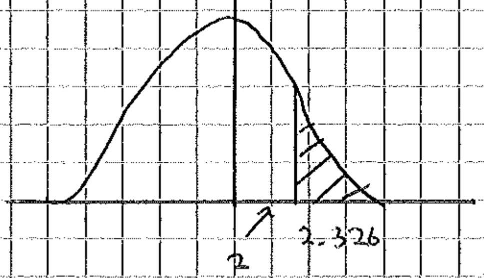
Stats 2 - January 2010

(1) $H_0: \mu = 45$ $n = 30$, so use Z

$H_1: \mu > 45$ (1 tailed test)

Test Statistic: $z = \dfrac{45.8 - 45}{\sqrt{4.8/30}} = 2$

Critical Value, 1%, 1 tailed = 2.3263



$2 < 2.326$

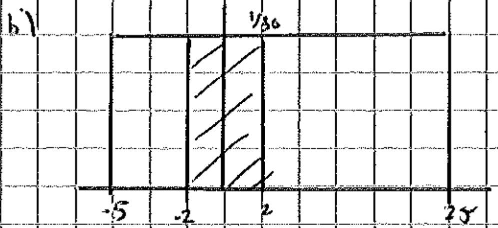
$\therefore$ Accept H_0

Not enough evidence at 1% level to support Roger's claim

(2) a) i) $E(T) = \frac{1}{2}(25 + -(5)) = 10$

ii) $Var(T) = \frac{1}{2}(25 - (-5))^2 = 75$

b)



$P(-2 < t < 2) = 4 \times \frac{1}{30} = \frac{4}{30}$

$\therefore P(|t| > 2) = 1 - \frac{4}{30} = \frac{13}{15}$

(3) $H_0: \mu = 190$

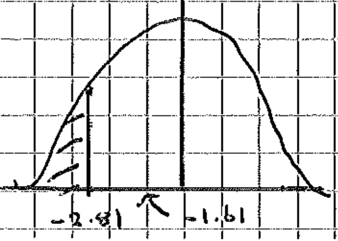
$H_1: \mu \neq 190$ (2 tailed test)

$\bar{x} = \frac{1840}{10} = 184$ $s^2 = \frac{1240}{9} = 137.\dot{7}$

$v = 10 - 1 = 9$

Test Statistic $= \dfrac{184 - 190}{\sqrt{137.7/10}} = -1.6164..$

Critical Value, $t_{(9)}$ 2%, 2 tailed $= \pm 2.821$



$-2.81 < -1.61$

$\therefore$ Accept H_0

Evidence suggests Lorraine is correct that there has been no significant change.

Assumption: The length of shots are normally distributed.

(4) a) H_0: No association between age + performance (Independent)

H_1: Association between age + performance (Non-Independent)

Observed

	Pass	Fail	
17-18	28	20	48
19-30	2	14	16
31-39	12	33	45
40-60	6	5	11
	48	72	120

Expected

	Pass	Fail
17-18	19.2	28.8
19-30	6.4	9.6
31-39	18	27
40-60	4.4	6.6

As Expected < 5
Need to combine final 2 rows

31-60	22.4	33.6

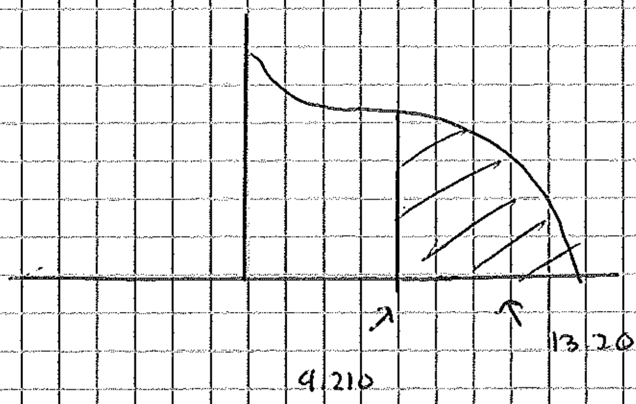
X_2 $\quad \frac{(O-E)^2}{E}$

	Pass	Fail
17-18	4.0333	2.6889
19-30	3.0256	2.0167
31-60	0.8643	0.5762
40-60	///	///

Test Statistic (Total X_2) = 13.20

Critical Value: $v = (3-1) \times (2-1) = 2$

$X^2_{(2)}$ 1% : 9.210



13.20 > 9.210

∴ Reject Ho.

Evidence at 1% level to support Julie's belief about association between age and passes

b) Observed: 28 passed, Expected: 19.2 passed

∴ More students passed than expected

⑤ a) Let X = number with disorder

$X \sim B(25, 0.7)$

$P(X > 15) = P(X \geq 16)$

Best to think at Y = number that don't have it

$Y \sim B(25, 0.3)$

$P(X \geq 16) = P(Y \leq 9) = 0.8106$ from tables

b) i) $X \sim Po(2.6)$

$P(X \leq 5) = 0.951$ from tables

ii) 49 per million = 4.9 per 100,000

$Y \sim P(4.9)$

$P(Y = 10) = \dfrac{e^{-4.4} \times (4.9)^{10}}{10!} = 0.0164$

iii) $T \sim Po(7.5)$

$P(T > 16) = 1 - P(T \leq 16)$

$= 1 - 0.9980 = 0.002$

⑥ a) i) $a = 25/63$ (prob sum to 1)

ii) $E(X) = 2.5$ (as distribution is symmetrical)

iii) $E(X^2) = 0^2 \times 1/252 + 1^2 \times 25/252 + 2^2 \times 100/252$

$+ 3^2 \times 100/252 + 4^2 \times 25/252 + 5^2 \times 1/252$

$= 125/18$

$\therefore Var(X) = \cancel{225/8}\ 125/18 - 2.5^2 = 25/36$

$\therefore sd(X) = \sqrt{25/36} = 5/6$

b) i) $E(\text{money}) = 40p \times 2/9 + (-50p) \times 5/9 + 60p \times 2/9$

$= 10p$ loss per game !

ii) After 100 games, expected loss = $100 \times 10p = £10$

(7) a) i) $s = \sqrt{\frac{93}{12}} = \sqrt{7.75}$

$\therefore s^2 = 7.75$

ii) $n = 12$, so $v = 11$

t value, $v = 11$, 80% CI, 2 tailed $= 1.363$

so CI $= 64.8 \pm 1.363 \times \sqrt{7.75}$

$= 64.8 \pm 3.79$

$= (61.0, 68.6)$ (3sf)

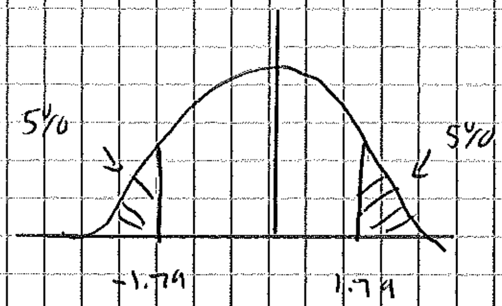
b) i) Symmetrical about mean $= (59.8, 69.8)$

ii) $64.8 + t\sqrt{7.75} = 69.8$

$t\sqrt{7.75} = 5$

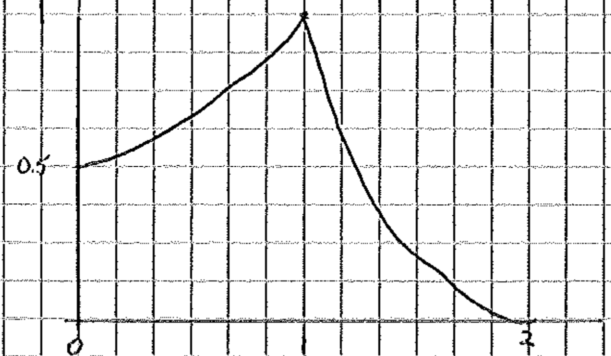
$t = 5/\sqrt{7.75} = 1.7960...$

From tables, $P(X > 1.7960) = 0.05$



$\therefore$ 90% confidence interval

(8)



b) $P(X \leq 1) = \int_0^1 \frac{1}{2}(x^2+1)\,dx$

$= \left[\frac{x^3}{6} + \frac{x}{2}\right]_0^1$

$= \frac{1}{6} + \frac{1}{2} - 0 = \frac{2}{3}$

c) $E(X^2) = \int_0^1 x^2 \, \frac{1}{2}(x^2+1) + \int_1^2 x^2(x-2)^2$

$= \int_0^1 \frac{1}{2}x^4 + \frac{1}{2}x^2 + \int_1^2 x^4 - 4x^3 + 4x^2$

$= \left[\frac{x^5}{10} + \frac{x^3}{6}\right]_0^1 + \left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3}\right]_1^2$

$= \frac{1}{10} + \frac{1}{6} + \left(\frac{32}{5} - 16 + \frac{32}{3}\right) - \left(\frac{1}{5} - 1 + \frac{4}{3}\right)$

$= \frac{4}{5}$

d) i) $Var(X) = E(X^2) - E(X)^2$

$\frac{499}{K} = \frac{4}{5} - \left(\frac{19}{24}\right)^2$

$\frac{499}{K} = \frac{499}{2880} \Rightarrow K = 2880$

ii) $E(5X^2 + 24X - 3)$

$= 5E(X^2) + 24E(X) - 3$

$= 5 \times \frac{4}{5} + 24 \times \frac{19}{24} - 3 = 20$

iii) $Var(12X - 5)$

$= 12^2 \, Var(X) = 144 \times \frac{499}{2880} = \frac{499}{20}$